Journal of Basic and Applied Engineering Research
Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 2, Number 13; April-June, 2015 pp. 1075-1080

http://www.krishisanskriti.org/jbaer.html

Analysis of Submerged Flexible Vegetation for Open Channel Flow

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Abstract—*The Vegetation of channels is a modern technique. It alters the structure of water flow, and helps to increase the bed roughness. It can help to dissipate incoming wave energy, regulate water levels, improve water quality and support recreational activities, whereas vegetation may reduce channel conveyance capacity. In this paper the deflection height of the flexible vegetation is obtained with the help of large-deflection cantilever beam theory. The flow regime can be divided into two layer, a bottom vegetation layer and an upper free layer for a uniform, steady, and fully developed turbulent open channel flow. In the bottom vegetation layer, the deflected plant's resistance is calculated for plant bending, which is more important than the resistance formula for erect rigid vegetation. In the upper free water layer, to obtain the zero velocity gradient at the water surface a new type of polynomial velocity distribution technique is suggested. Two layer analytical model suitable for both flexible and rigid vegetation was established in this paper. This analysis shows that among various hydraulic parameters and vegetation parameters the drag coefficient is crucial for a proper prediction.*

Keywords: *Deflection of flexible vegetation, large-deflection cantilever beam theory, velocity distribution, vegetation swing, drag coefficient*

1. INTRODUCTION

Vegetation-fluid interaction has been the subject of numerous studies involving laboratory, numerical, analytical, and field investigations. Vegetation is divided into two types depending on its rigidity, (i) rigid, (ii) flexible. Much of the knowledge gained has been gleaned from laboratory tests with various conditions including emergent and submerged flow through rigid or flexible vegetation.These studies used either artificial roughness elements or real specimens for the investigation of various properties like turbulence features, resistance induced by the vegetation system.

Velocity distribution of open channel flow is no doubt a significant problem. Numerical and analytical solutions widely used, for rigid vegetation, in determining the velocity distribution. A numerical can give almost accurate results and different numerical strategies were adopted. But for practical application, it is not convenient as because this numerical solution has a high calculation cost. That's why; a simply analytical solution is a good substitution for a preliminary estimate. The eddy viscosity model, the missing length models have been presented to predict the vertical profile of stream-wise velocity. According to different force balance conditions, the flow region has been vertically divided into two, three or even four layers too. In this paper velocity profile was assumed to be uniform in the vegetation layer and a logarithmic velocity profile for the upper layer.

The experimental data of Kubark*et al* 2008 was used to measure the deflection height of vegetation and calculate the error comparing with the results of that paper.

2. DYNAMIC ANALYSIS

2.1 Deflection of flexible vegetation

The approach proposed here is based on the theory of large deflection cantilever beams, and thus better approximates reality. If bending occurs, the model of flow past a cylinder must be adjusted. Assume that each small plant segment can be modelled as an inclined cylinder in the flow and that different portions of the plant do not affect each other, so that a dynamic analysis can be conducted. According to the theory of large deflection cantilever beams (Chen 2010), assuming that the beam material remains linearly elastic, the relationship between the bending moment and the beam deformation reads as

$$\frac{\frac{d^2x}{dy^2}}{[1+(\frac{dy}{dx})^2]^{3/2}} = \frac{M\,(y)}{EI} \qquad (1)$$

Here x and y are coordinates with y parallel to the original beam M is the bending moment (Nm), E is the modulus of elasticity of the material (Nm^{-2}) and I is the moment of inertia of the cross-sectional area of the beam about the axis of bending (m^4).

Denoting$z = \frac{dx}{dy}$,

Chen (2010) converted the equation to

$$\frac{dz}{[1+z^2]^{3/2}} = \frac{M\,(y)}{EI}dy \tag{2}$$

By integrating we obtain

$$\frac{z}{\sqrt{1+z^2}} = \int_0^y \frac{M\,(y)}{EI}dy = G(y) \tag{3}$$

Denoting s as the distance measured along the bending curve,

$$\frac{ds}{dy} = [1+z^2]^{1/2} \tag{4}$$

Now Eq. (3) can be converted to

$$\frac{ds}{dy} = \frac{1}{\sqrt{1-G^2(y)}} \tag{5}$$

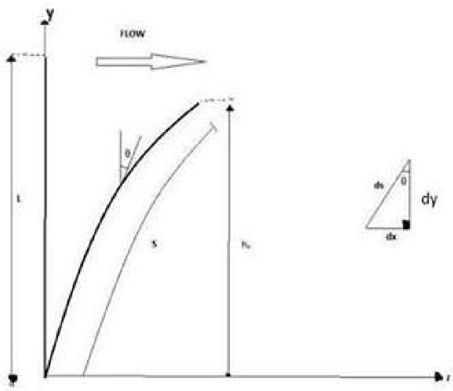

Fig. 1: Deflection of flexible vegetation

From $z = \frac{dx}{dy}$, another equation for variable x can be obtained, which reads

$$\frac{dx}{dy} = \frac{G(y)}{\sqrt{1-G^2(y)}} \tag{6}$$

The curve length of the beam can be calculated as

$$s(hv) = \int_0^{hv} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy \tag{7}$$

Employing the notation that in flowing water,the total load P is uniformly distributed over a single stem and normal to axis y,the bending moment function can be expressed as

$$M(y) = \frac{P(hv-y)^2}{2hv} \tag{8}$$

Vegetation height is h_v. G(y) is founded to be

$$G(y) = \frac{{}^{p}/_{hv}}{2EI}\left(\frac{y^3}{3} - hvy^2 + hv^2y\right)$$

$$= \frac{p}{2EI}\left(\frac{y^3}{3hv} - y^2 + hvy\right) \tag{9}$$

Therefore the governing equations are

$$\frac{ds}{dy} = \frac{1}{\sqrt{1-(\frac{p}{2EI})^2(\frac{y^3}{3hv}-y^2+hvy)^2}} \tag{10}$$

and

$$\frac{dx}{dy} = \frac{(\frac{p}{2EI})(\frac{y^3}{2hv}-y^2+hvy)}{\sqrt{1-(\frac{p}{2EI})^2(\frac{y^3}{3hv}-y^2+hvy)^2}} \tag{11}$$

For uniform, steady open channel flow, the total load P exertedby flowing water over a single stem is

$$P = \frac{1}{2}\rho C_D D h_v U_v^2 \tag{12}$$

Where ρ is the water density, $C_D$is the drag coefficient, D isthe frontal-projected width of the stem, and$U_v$is the velocityaveraged over the vegetation layer.

For a uniform, steady open channel flow with dense vegetation, the bottom shear can be ignored and it is proposed that

$$U_v = \sqrt{\frac{2giH}{C_D m D h_v}} \tag{13}$$

Where g is the gravitational acceleration, i is the energy slope which equals the slope of channel bed, H is the total flow depth, and m is the vegetation density, which is defined as the number of stem per unit bed area.

Assuming the drag coefficient C_D for a single stem and the vegetation cluster are the same. It may be different with the practical situation but it is suitable for a simplified theoretical analysis. Substituting Eq. (13) into Eq. (12) gives the total load P.

$$P = \frac{\rho giH}{m} \tag{14}$$

It is the intuitive to think that the total force on a single stem is a function of the stem properties (its deflection height, h_v, and frontal-projected width, D, as that shown in Eq.(12);however Eq. (14)is not a function of these stem properties. Although, for different experimental cases with various stem properties (h_v, D), the resistance of open channel flow will change, and the depth of the steady and uniform flow, H, will change concomitantly. So Eq. (14) includes the effect of stem properties completely.

Substituting Eqs. (11) and (14) into Eq. (7) gives the curve length of the single stem:

$$S(hv) = \int_0^{hv} \frac{1}{\sqrt{1-\left[\left({}^{\rho giH}/_{2mEI}\right)\left(\frac{y^3}{3hv}-y^2+hvy\right)\right]}}dy \tag{15}$$

To find the deflection height, h_v, Eq. (15) can be solved numerically. With an initial given h_v, G(y) can be determined from Eq. (9). $dx/dy$is then obtained from Eq. (11). The arc length $S(h_v)$ is obtained by substituting Eqs. (11) and (14) into Eq. (7). If $|S(h_v)-L| > \varepsilon$, a new value should be assumed until $|S(h_v)-L| < \varepsilon$ where ε is the computational accuracy determining the accuracy of iterations.

2.2 Reynolds stress distribution

For a uniform, steady, and fully developed turbulent open channel flow with flexible vegetation, the flow regime can be divided into a bottom vegetation layer and an upper free water layer, as illustrated in Fig.2.

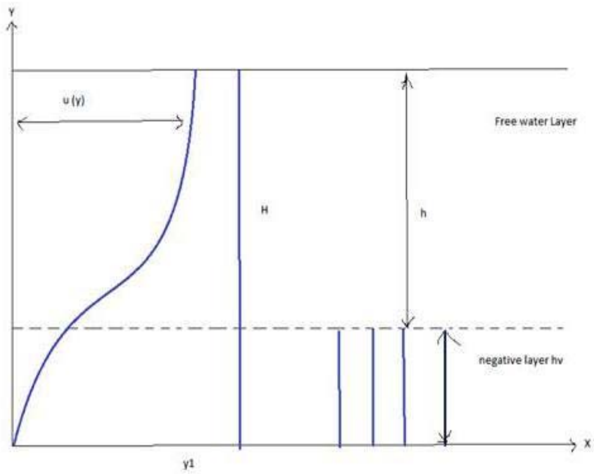

Fig. 2: Open channel flow of submerged vegetation

In vegetation layer, Dijkstra and Uittenbogaard (2010) collated data of Reynolds stress and found that data for rigid and flexible stems were similar. In that the Reynolds stress conforms to an exponential profile and the peak value occurs near the top of vegetation. On this basis, the vertical distribution of the Reynolds stress can be expressed as

$$\tau = -\rho\overline{u'w'} = -\rho\overline{u'w'}\,|_{z=h_v}\; e^{\alpha(z-h_v)} \qquad (16)$$

whereα is a constant and $\bar{u}$ and $\bar{w}$ are the temporal fluctuations from the means of longitudinal and vertical velocities respectively.

From the momentum balance of the flow above the vegetation, the interfacial shear stress between vegetation and free water layers can be obtained by (Yang and Choi 2010)

$$-\rho\overline{u'w'}\,|_{z=h_v} = \rho gi\left(H-h_v\right) = \rho u_*^2 \qquad (17)$$

where $u_* = \sqrt{gih}$ the shear velocity at the top of the plant, h is the water depth above the vegetation top(i.e.h=H-h_v). For the upper free water layer, the eddy viscosity model of Boussinesq was employed to describe the Reynolds shear stress

$$\tau = \rho v_t \frac{du}{dy} \qquad (18)$$

Where $v_t$is the eddy viscosity.

2.3 Two-layer Model

In the vegetation layer, the viscosity shear stress was omitted and only the Reynolds shear stress was considered. Considering the force balance between the Reynolds shear stress, gravity component, and resistance force imparted by vegetation, the momentum equation can be deduced as

$$\frac{\partial\tau}{\partial y} + \rho gi - \frac{\partial F_n}{\partial y} = 0 \qquad (19)$$

From Eqs. (16) and (17), the Reynolds shear stress gives

$\tau = \rho ghie^{\alpha(y-h_v)}$ (20)For flexible vegetation, there are two types of force exerted by flowing water on a small plant element ds, as shown in Fig. 3 (taking a cylinder, for example): the drag force dFD, which isnormal to the plant stem, and the friction force dFf, which isalong the plant. These forces (per unit area of the bed) can be calculated by the method proposed by Bootle (1971):

$$dF_D = 1/2mC_D\rho(u\cos\theta)^2 A_f = 1/2mC_D\rho(u\cos\theta)^2 D\,ds \qquad (21)$$

$$dF_D = \frac{1}{2mC_f\rho(u\sin\theta)^2 A_s} = \frac{1}{2mC_f\rho(u\sin\theta)^2\pi D}ds \qquad (22)$$

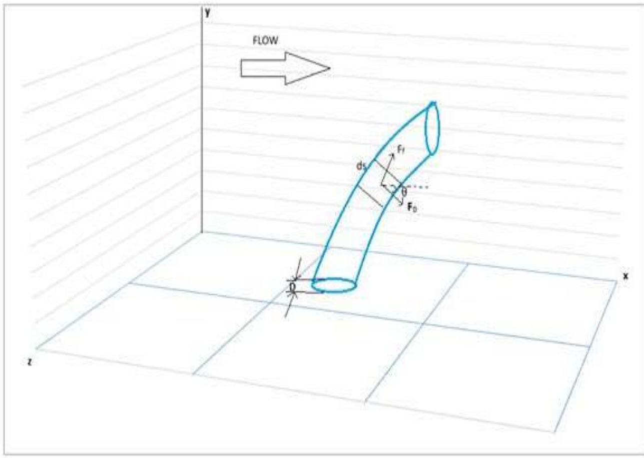

Fig. 3: View of a single bending stem

where*Cf*is the friction coefficient,*Af*is the frontal area of thestem, *As* is the surface area of the stem,*Cp*is the perimeter of thestem cross section. For circular cylinders, the frontal-projected width of the stem, *D*, is equal to the stem diameter and*Cp*is the perimeter of the stem cross section. For non-circular cylinders, the calculations of*Af*and *As* in Eqs. (21) and (22) should be changed via the geometry of the stem cross section. According to Newton's Third Law, the acting force and reacting force are equal and opposite in direction, and the resistance force imparted by vegetation can therefore be described by the above expressions.

The geometric relationship gives

$$ds = \frac{dy}{\cos\theta} \qquad (23)$$

From Eq. 3 and Fig. 1

$$G(y) = Sin\theta \quad (24)$$

$$\sin\theta = (\frac{p}{2EI})(\frac{y^3}{3hv} - y^2 + hvy) \quad (25)$$

Projecting $dF_D$and dF_fon the *x*-axis gives the resultant forcedFx:

$$dF_x = dF_D cos\theta + dF_g sin\theta \quad (26)$$

$$dF_x = \frac{1}{2} mC_D \rho (ucos\theta)^2 D\ ds\ .cos\theta + \frac{1}{2} mC_f \rho (usin\theta)^2 \pi D\ ds\ .sin\theta$$

$$dF_x = \frac{1}{2} m\rho u^2 D\{C_D \left[cos^3\theta.\frac{dy}{cos\theta}\right] + \pi C_f [sin^3\theta \frac{dy}{cos\theta}] \quad (27)$$

Substituting Eqs. (12),(21), (22), (24) and (25) into Eq. (27) gives the resultant force of the vegetation in the horizontal direction

$$\frac{\partial F_x}{\partial y} = \frac{1}{2} m\rho u^2 D\{C_D \left[1 - \left({\rho giH}/{2mEI}\right)^2 \left(\frac{y^3}{3hv} - y^2 + hvy2 + C_D \pi \frac{[\left({\rho giH}/{2mEI}\right)\left(\frac{y^3}{3hv} - y^2 + hvy\right)]^3}{\sqrt{1 - [\left({\rho giH}/{2mEI}\right)\left(\frac{y^3}{3hv} - y^2 + hvy\right)]^2}} \quad (28)$$

From Eq. (28), one can find that the resistance force of the bending vegetation and that of the erect one are significantly different. The drag force of erect vegetation is $0.5C_D mD\rho u^2$(Hoerner 1965). In addition to the drag force component, the friction force component was included in the resistance formula of bending vegetation.

Substituting Eqs. (20) and (27) into Eq. (19) to solve the momentum equation, the vertical velocity distribution in the vegetation layer is obtained

$$u = \frac{2gi\left[\alpha h e^{\alpha(y-h_v)} + 1\right]}{\sqrt{m\left\{C_D D\left[1 - \left(\frac{\rho giH}{2mEI}\right)^2\left(\frac{y^3}{3h_v} - y^2 + yh_v\right)^2\right] + \frac{C_f C_p\left[\left(\frac{\rho giH}{2mEI}\right)\left(\frac{y^3}{3h_v} - y^2 + yh_v\right)\right]^3}{\sqrt{1 - \left[\left(\frac{\rho giH}{2mEI}\right)\left(\frac{y^3}{3h_v} - y^2 + yh_v\right)\right]^2}}\right\}}} \quad (29)$$

When y= h_v, the velocity at the top of the plant u_v is

$$u_v = \frac{2gi(\alpha h + 1)}{\sqrt{m\left\{C_D D\left[1 - \left(\frac{\rho giHh_v^2}{6mEI}\right)^2\right] + \frac{C_f C_p\left(\frac{\rho giHh_v^2}{6mEI}\right)^3}{\sqrt{1 - \left(\frac{\rho giHh_v^2}{6mEI}\right)^2}}\right\}}} \quad (30)$$

For the upper free water layer, the viscosity shear stress can also be omitted, and only the Reynolds shear stress needs to be considered. The Boussinesq eddy-viscosity concept is applied to solve the Reynolds shear stress and the governing equation of the upper free water layer reads

$\rho gi(H - y) = \rho \frac{du}{dy} V_t$(31)IntegratingEq.(31)gives

$$u = \frac{gi}{V_t}(Hy - \frac{1}{2}y^2) + C_1 \quad (32)$$

Where C_1 is the integrating constant, with u=u_v, at y= h_v, it follows that

$$C_1 = u_v - \frac{gi}{V_t}\left(Hhv - \frac{1}{2}hv^2\right) \quad (33)$$

The vertical velocity distribution for the upper free water layer is

$$u = \frac{gi}{2v_t}\left(-y^2 + 2Hy - 2Hh_v + h_v^2\right) + u_v, y > h_v \quad (34)$$

Huai*et al.* (2009a) proposed a vertical velocity distribution for the upper free water layer under the condition of steady and uniform flow

$$u = \left(\frac{H}{h}\right)\frac{u_*}{k}\ln\frac{z}{h_v} + u_v, y > h_v \quad (35)$$

Where k = 0.41

Integrating Eq. (34) from the top of the vegetation to the water surface, and then dividing by *h*, gives the velocity averaged over the free water layer U_{w1}

$$U_{w1} = \frac{1}{h}\int_{hv}^{H} u(y)\, dy = \frac{gih^2}{3V_t} + u_v \quad (36)$$

In the same way, integrating Eq. (35) from the top of the vegetation to the water surface, and then dividing by h, gives the velocity averaged over the free water surface, and then dividing by h, gives the velocity averaged over the free water layer U_{w2}

$$U_{w2} = \frac{1}{h}\int_{hv}^{H} u(y)\, dy = \frac{H\sqrt{giH}}{Kh^2}(Hl_n\frac{H}{hv} - h) + u_v \quad (37)$$

Making Eqs. (36) and (37) equals, the eddy viscosity can be expressed as

$V_t = \frac{K(gi)^{\frac{1}{2}}h^{\frac{7}{2}}}{3H^2 l_n\left(\frac{H}{hv}\right) - 3Hh}$(38)The vertical velocity distribution for

the upper free water layer is obtained by substituting Eq. (38) into Eq. (34):

$$u = \frac{3H^2 l_n\left(\frac{H}{hv}\right) - 3Hh}{Kh^{\frac{7}{2}}} \times (-y^2 + 2Hy - 2Hhv + hv^2) + uv \quad (39)$$

From Eq. (39), one can find that the velocity gradient is zero at the water surface:

$$\frac{du}{dy}|_{y=H} = 0 \quad (40)$$

3. RESULTS AND ANALYSIS

3.1 Deflection heights

Deflection heights is solved numerically from the Eq, (15), which is solved through an algorithm in MATLAB software. These results are compared with the experimental data of Kubrak*et al.* (2008) and the results. In the experiment of Kubrak*et al.* (2008), two horizontal components of mean velocity (longitudinal and transversal) were measured in a 16 m long, 0.58 m wide glass-walled flume.

Table 1: Demand and deficit for users in command

Case	Initial Plant Height (m)	Measured deflected height (m)	Calculated deflected height (m)	Calculation error (m)
1.1.1	0.165	0.163	0.163	0.000
1.1.2	0.165	0.163	0.163	0.000
1.1.3	0.165	0.164	0.164	0.000
1.1.4	0.165	0.164	0.164	0.000
1.2.1	0.165	0.161	0.161	0.000
1.2.2	0.165	0.162	0.161	-0.001
1.2.3	0.165	0.161	0.161	0.000
1.2.4	0.165	0.162	0.162	0.000
2.1.1	0.165	0.154	0.153	-0.001
2.1.2	0.165	0.154	0.154	0.000
2.1.3	0.165	0.155	0.154	-0.001
2.2.1	0.165	0.132	0.131	-0.001
2.2.2	0.165	0.131	0.131	0.000
2.2.3	0.165	0.133	0.132	-0.001
3.1.1	0.165	0.151	0.151	0.000
3.1.2	0.165	0.152	0.151	-0.001
3.1.3	0.165	0.153	0.153	0.000
3.2.1	0.165	0.131	0.131	0.000
3.2.2	0.165	0.139	0.138	-0.001
4.1.1	0.165	0.151	0.151	0.000
4.1.2	0.165	0.153	0.153	0.000
4.1.3	0.165	0.157	0.156	-0.001
4.2.1	0.165	0.138	0.138	0.000
4.2.2	0.165	0.142	0.142	0.000
4.2.3	0.165	0.143	0.142	-0.001

Calculated error = (calculated deflected height – measured deflected height)

From Table 1 we can see that the theoretical results have good similarity with the experimental data (Kubrak et al. 2008). In some cases calculated data were slightly smaller than the experimental data. This was happened because of interaction among bending plants, which has not been considered in the theoretical analysis. So now the deflection height is a known parameter in the model verification to obtain the velocity distribution.

3.2 Determination of coefficient.

There are three parameters the drag coefficient C_D, friction coefficient C_f, and constantα all are required for the two-layer model.

3.2.1 Drag coefficient C_D heights.

The local drag coefficient C_D is determined as that suggested by Schlichting (1979)

$$C_D = 3.07R_D^{-0.168},\ R_D < 800$$
$$C_D = 1.0,\ 800 \le R_D \le 800 \quad (41)$$
$$C_D = 1.2,\ 8000 \le R_D \le 10^5$$

Where R_D denotes the drag Reynolds number, which can be calculated as

$$R_D = \frac{uDcos\theta}{\gamma} \quad (42)$$

Here u is the flow velocity at depth y and υ is the kinematic viscosity of water. From Eq. (41), When R_D<800, the local C_D varies with the flow velocity. According to the experimental results presented by Kubrak*et al*. (2008. We concluded that R_D is less than 800 in the vegetation layer, and C_D thus varies between 1 and 1.5.

3.2.2 Friction coefficient C_f.

The local friction coefficient C_f is determined suggested by Suryanarayana and Arici (2002)

$$C_f = \frac{1.328}{R_F^{0.5}},\ R_F < 5\times10^5$$
$$C_f = \frac{0.074}{R_F^{0.5}} - \frac{1740}{R_F},\ 5\times10^5 < R_F < 10^7 \quad (43)$$
$$C_f = \frac{0.455}{\log(R_F)^{2.584}} - \frac{1700}{R_F},\ 10^7 < R_F$$

Where R_F denotes the friction Reynolds number, which can be calculated as

$$R_F = \frac{su\sin\theta}{\upsilon} \quad (44)$$

Here s is the bending curve length measured from the base of the plant to the point R_F to be calculated. In the analytical model, a bulk friction coefficient instead of the local one is needed. In this model, the bulk friction coefficient C_f is taken as 0.4.

3.2.3 Coefficient α.

The value of α has been related to the hydraulic and vegetation characteristics. From Kubrak et al. (2008) a suitable value of α can be obtained from the expression is

$$\alpha = \sqrt{\frac{C_D mD}{0.03(H - h_v)}} \quad (45)$$

The result shown that α varies with drag coefficient, vegetation frontal projected width, the total flow depth and the deflection height of plants. The fitting of α is shown in Fig. 4.

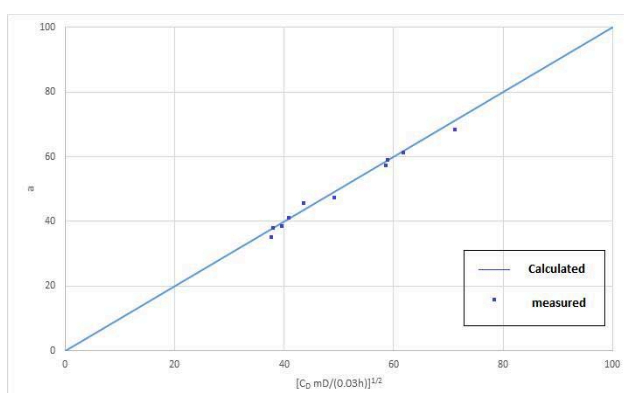

Fig. 4: The fir of alpha

3.2.4 Model verification

The two layer model of flexible vegetation was applied to the experimental data suggested by Kubrak*et al.* (2008) for the model verification. Three sets of parameter were taken from Kubrak*et al.* (2008). The parameters are given in the Table 2.

Table 2: Parameters of experiments

Run	**1.2.1**	**2.2.1**	**4.1.1**
Cross-sectional shape	**Ellipse**	**Ellipse**	**Ellipse**
I	0.0174	0.0174	0.0087
A	84.15	36.97	34.88
C_D	1.4	1.4	1.4
D (m)	0.00095	0.00095	0.00095
C_P (m)	0.0026067	0.0026067	0.0026067
H (m)	0.2236	0.2131	0.2421
L (m)	0.165	0.165	0.165
h_v(m)	0.161	0.132	0.151
EI (Hm^2)	5.81E-05	5.81E-05	5.81E-05
m (stems/m^2)	10000	2500	2500

The velocity comparison is presented in Fig. 5.

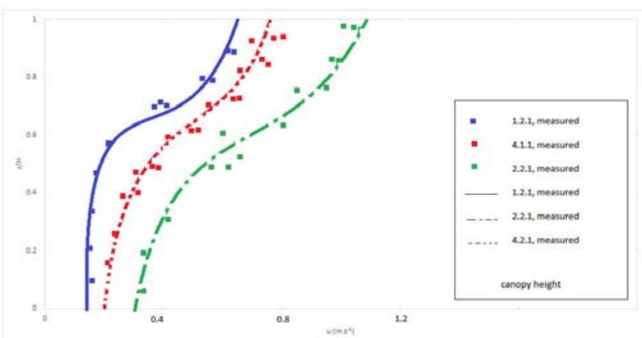

Fig. 5: Comparison of measurement and analytical results

It can be concluded that the theoretical results are in good agreement with the experimental data, which means that the theoretical formula can be applied to predict the vertical velocity distribution of open channel flow with submerged flexible and rigid vegetation.

4. CONCLUSIONS

A two-layer analytical model suitable for open channel flow with both flexible vegetation and rigid vegetation was established in this paper. Compared with other methods, such as numerical solutions, laboratory measurements, and field investigations, this analytical model is convenient to use in practical applications, especially for the purpose of a primary evaluation. Adopting the theory of a large-deflection cantilever beam, the deflection height of the flexible vegetation can be obtained. Based on the momentum balance in the vegetated layer, vertical profile of the stream-wise velocity in that layer was deduced.

This model can be used to artificial and natural channels in which the flow is approximately uniform and steady, and basic hydraulic parameters (e.g. energy slope*i*and water depth *H*) andvegetation parameters (e.g. the vegetation density *m*, deflectionheighth_v, module of elasticity *E*, and cross-sectional shape of thestems, which is necessary to obtain the frontal-projected width *D*, moment of inertia *I* and perimeter$C_p$of the cross sectionof stems) are required. The sensitivity analysis shows that the determination of those parameters, especially the drag coefficient $C_D$is crucial for a sound prediction.

Although there are many studies on velocity distribution of open channel flow with submerged flexible plants, data of *E* (i.e. module of elasticity of the material) were rare. For this reason, Kubrak *et al.* 2008 is adopted for verification, and the application with a wider range of *E* shouldbe checked. For the limited cases studied here, the bulk friction coefficient$C_f$was taken as 0.4. But one should notice thatfor different experimental conditions and materials, the frictioncoefficient may be different.

5. ACKNOWLEDGEMENT

The work presented in this paper would not been possible without the support that I received from my supervisor Prof. KaushikBandyopadhyay. I am thankful to the Department of Construction Engineering, Jadavpur University for providing me with the facilities needed for the completion of my project.

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